

# Resonant Circuits

Analysis of Series & Parallel R-L-C Configurations

A Comprehensive Guide to Derivations, Parameters, and Solved Examples

# Understanding Electrical Resonance

## What is Resonance?

- Occurs in AC circuits containing both inductors (L) and capacitors (C).
- At a specific frequency, inductive reactance ( $X_L$ ) equals capacitive reactance ( $X_C$ ).
- The reactive effects cancel each other out, making the total circuit input impedance purely resistive.
- Voltage and total current become perfectly in phase (Power Factor = 1).

## Fundamental Mathematical Conditions

- Inductive Reactance:  $X_L = \omega L$
- Capacitive Reactance:  $X_C = 1 / (\omega C)$
- Resonance Balance Condition:  $X_L - X_C = 0 \Rightarrow X_L = X_C$
- The circuit behaves as a pure resistor (R), minimizing or maximizing current depending on configuration.

# Series R-L-C Circuit Under Resonance

## Impedance and Current Characteristics

- Total Impedance:  $Z = R + j(X_L - X_C)$
- At Resonance:  $X_L = X_C$ , so  $Z = R$  (Minimum Impedance)
- Current Response:  $I = V / Z = V / R$  (Maximum Current)
- Phase Angle:  $\theta = 0^\circ$ , meaning input voltage and current are perfectly in phase.
- Acceptor Circuit: Often called an acceptor circuit because it passes maximum current at resonance.

## Derivation of Resonant Frequency ( $\omega_0$ )

1. Set  $X_L = X_C$
2. Substitute reactances:  $\omega_0 * L = 1 / (\omega_0 * C)$
3. Solve for  $\omega_0^2$ :  $\omega_0^2 = 1 / (L * C)$
4. Angular Resonant Frequency:  $\omega_0 = 1 / \sqrt{L * C}$  (rad/s)
5. Cyclic Resonant Frequency:  $f_0 = 1 / (2 * \pi * \sqrt{L * C})$  (Hz)

# Series Parameters: Bandwidth & Quality Factor

## Cut-off Frequencies & Bandwidth (BW)

• **Half-Power / Cut-off Points:** Frequencies  $\omega_1$  and  $\omega_2$  where power drops to half maximum, and current drops to  $I_{\max} / \sqrt{2}$ .

**Lower Cut-off ( $\omega_1$ ):**  $\omega_0 - R/(2L)$

**Upper Cut-off ( $\omega_2$ ):**  $\omega_0 + R/(2L)$

**Bandwidth Formula:**  $BW = \omega_2 - \omega_1 = R / L$  (rad/s)

**In terms of Cyclic Frequency:**  $BW$  (Hz) =  $f_2 - f_1 = R / (2\pi L)$

## Quality Factor (Q-Factor)

Definition: Ratio of reactive power to active power at resonance. Measures sharpness of resonance curve.

Mathematical Expressions:

$$Q = \omega_0 * L / R = 1 / (\omega_0 * C * R)$$

Substituting  $\omega_0$  gives alternative form:

$$Q = (1 / R) * \sqrt{L / C}$$

Relationship with Bandwidth:

$$Q = \omega_0 / BW = f_0 / BW$$

High Q implies a narrow, highly selective response curve.

# Parallel R-L-C Circuit Under Resonance

## Admittance and Current Dynamics

- Total Admittance:  $Y = (1/R) + j(\omega C - 1/(\omega L))$
- At Resonance: Susceptance cancels out ( $\omega C = 1/(\omega L)$ )
- Resulting Admittance:  $Y = 1 / R$  (Minimum Admittance)
- Impedance:  $Z = R$  (Maximum Impedance)
- Total Line Current:  $I = V * Y = V / R$  (Minimum Line Current)
- Rejector Circuit: Rejects the resonant frequency by presenting maximum impedance.

## Resonant Frequency & Parameters

Resonant Frequency (Ideal Parallel):

$$\omega_0 = 1 / \sqrt{L * C} \quad (\text{Identical to series form})$$

Quality Factor (Q) for Parallel:

$$Q = R / (\omega_0 * L) = \omega_0 * R * C = R * \sqrt{C / L}$$

Note: Parallel Q expression is inverted relative to series.

Bandwidth (BW) for Parallel:

$$BW = \omega_0 / Q = 1 / (R * C) \quad (\text{rad/s})$$

$$BW \text{ (Hz)} = 1 / (2 * \pi * R * C)$$

# Direct Comparison: Series vs. Parallel Resonance

| Parameter / Property              | Series R-L-C Circuit                      | Parallel R-L-C Circuit (Ideal)            |
|-----------------------------------|-------------------------------------------|-------------------------------------------|
| Resonant Frequency ( $\omega_0$ ) | $1 / \sqrt{LC}$                           | $1 / \sqrt{LC}$                           |
| Impedance at Resonance            | Minimum ( $Z = R$ )                       | Maximum ( $Z = R$ )                       |
| Current at Resonance              | Maximum ( $I = V / R$ )                   | Minimum ( $I = V / R$ )                   |
| Circuit Behavior                  | Acceptor (passes $\omega_0$ line current) | Rejector (blocks $\omega_0$ line current) |
| Quality Factor (Q)                | $(1 / R) * \sqrt{L / C}$                  | $R * \sqrt{C / L}$                        |
| Bandwidth (BW in rad/s)           | $R / L$                                   | $1 / (R * C)$                             |

# Solved Example 1: Series R-L-C Circuit

## Problem Statement

A series R-L-C circuit has the following component values:

- Resistance (R) = 10  $\Omega$
- Inductance (L) = 0.1 H
- Capacitance (C) = 100  $\mu\text{F}$  ( $100 \times 10^{-6}$  F)
- Connected Source Voltage = 100V AC

Calculate:

1. Resonant Frequency ( $f_0$ )
2. Quality Factor (Q)
3. Bandwidth (BW in Hz)
4. Current at Resonance ( $I_{\text{max}}$ )

## Step-by-Step Solution

### 1. Resonant Frequency:

$$\omega_0 = 1 / \sqrt{L \cdot C} = 1 / \sqrt{0.1 \cdot 100 \times 10^{-6}} = 316.23 \text{ rad/s}$$

$$f_0 = \omega_0 / 2\pi = 316.23 / 6.283 = 50.33 \text{ Hz}$$

### 2. Quality Factor:

$$Q = (1/R) \cdot \sqrt{L/C} = (1/10) \cdot \sqrt{0.1 / 100 \times 10^{-6}} = 3.16$$

### 3. Bandwidth:

$$\text{BW (Hz)} = f_0 / Q = 50.33 / 3.16 = 15.92 \text{ Hz}$$

### 4. Resonant Current:

$$I_{\text{max}} = V / R = 100\text{V} / 10 \Omega = 10 \text{ A}$$

# Solved Example 2: Parallel R-L-C Circuit

## Problem Statement

An ideal parallel R-L-C circuit has the following values:

- Resistance (R) = 10,000  $\Omega$  (10 k $\Omega$ )
- Inductance (L) = 2 mH ( $2 \times 10^{-3}$  H)
- Capacitance (C) = 10 nF ( $10 \times 10^{-9}$  F)
- Connected Source Voltage = 50V AC

Calculate:

1. Resonant Frequency ( $f_0$ )
2. Quality Factor (Q)
3. Bandwidth (BW in kHz)
4. Total Line Current at Resonance

## Step-by-Step Solution

### 1. Resonant Frequency:

$$\omega_0 = 1 / \sqrt{L \cdot C} = 1 / \sqrt{2 \times 10^{-3} \cdot 10 \times 10^{-9}} = 223,606.8 \text{ rad/s}$$

$$f_0 = \omega_0 / 2\pi = 35,588.1 \text{ Hz} \approx 35.59 \text{ kHz}$$

### 2. Quality Factor:

$$Q = R \cdot \sqrt{C/L} = 10,000 \cdot \sqrt{10 \times 10^{-9} / 2 \times 10^{-3}} = 22.36$$

### 3. Bandwidth:

$$\text{BW (kHz)} = f_0 / Q = 35.59 \text{ kHz} / 22.36 = 1.59 \text{ kHz}$$

### 4. Resonant Line Current:

$$I_{\min} = V / R = 50\text{V} / 10,000 \Omega = 5 \text{ mA (0.005 A)}$$